Fermi Liquid without Quasiparticles and Electron Spectral Functions of Two-Dimensional High-T_c Superconductors

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Abstract

Properties of strongly correlated two-dimensional (2D) electron systems in solids are studied on the assumption that these systems undergo a phase transition, called fermion condensation, whose characteristic feature is flattening of the electron spectrum $\epsilon(\mathbf{p})$. Unlike the previous models in the present study, the decay of single-particle states is properly taken into account. Remarkably, the value of the topological charge ($N = 1/2$) remains unchanged, supporting the view that systems with a fermion condensate form a separate class of Fermi liquids. Results of calculations are found to be in qualitative agreement with ARPES data.

The single-particle dynamics of Fermi systems at near zero temperatures T is known to depend crucially on the index ν, that characterizes the imaginary part of the mass operator, $\mathrm{Im}\,\Sigma(\varepsilon \to 0) \sim \varepsilon^\nu$, the energy ε being measured from the chemical potential μ. In ordinary homogeneous Fermi liquids such as nuclear matter and liquid ^{3}He, where the exclusion principle "leads the dance", the index ν equals 2, and the Fermi liquid can be treated as a gas of interacting "immortal" quasiparticles, the cornerstone of standard Fermi liquid theory (SFLT) [1]. After many successful years, SFLT is currently encountering serious difficulties in treating normal states of 2D high-T_c superconductors. The analysis of ARPES data shows that even around the diagonals of the Brillouin zone, the index ν is unity [2,3], while in the immediate vicinity of the van Hove points (vHP), the sharp ARPES peaks disappear altogether [4–6].

We propose that solution of this challenging problem is associated with fermion condensation [7–17], a novel phase transition that generates a group of dispersionless states, called the fermion condensate (FC), whose energies coincide with μ. States with the FC have been uncovered [7] as unconventional solutions of equations of Landau theory at $T = 0$. To gain insight into the problem, consider the Landau formula for the variation of the ground state energy E_0

$$\delta E_0 = \sum_{\mathbf{p}} \epsilon(\mathbf{p}; n_F)\, \delta n(\mathbf{p}) + \frac{1}{2} \sum_{\mathbf{p}_1, \mathbf{p}_2} \Gamma(\mathbf{p}_1, \mathbf{p}_2; n_F)\, \delta n(\mathbf{p}_1)\, \delta n(\mathbf{p}_2)\ , \tag{1}$$

applicable if the change of the quasiparticle momentum distribution $n_F(\mathbf{p}) = \theta(\mu - \epsilon(\mathbf{p}))$ is rather small. The single-particle (SP) spectrum $\epsilon(\mathbf{p})$ and the scattering amplitude $\Gamma(\mathbf{p}_1, \mathbf{p}_2)$ are to be evaluated for the Landau state. The formula (1) is appropriate for homogeneous systems, where $\mathbf{p}$ is the momentum, as well as for finite ones, where $\mathbf{p}$ stands for a set of relevant SP quantum numbers. The crucial point is that due to the discreteness of the SP spectra, low-lying states of finite systems, adjacent to the ground state, do not practically damp. This situation persists until the excitation energy ε attains values $\sim \varepsilon_F^0/\sqrt{A}$, ($\varepsilon_F^0$ is the Fermi energy, A is the particle number), above which the SP widths $\gamma_f(\varepsilon)$, calculated with incorporation of broadening of energy δ-functions, exceed the distance between the SP

levels. Then the widths $\gamma_f(\varepsilon)$ and $\gamma(\varepsilon)$, inherent in infinite systems, rapidly approaches each other. We infer that in search for a new ground state of a large system, the momentum representation can be employed, while damping effects can be ignored. At the same time, in dealing with the spectral functions at realistic energies, damping effects should be properly taken into account.

As seen from Eq.(1), the Landau state holds until the necessary stability condition (NSC), requiring the positivity of the l.h.s. of Eq. (1) at any admissible variation of n_F, is violated, and thereby the minimum of $E_0(n)$ moves away from n_F. A prerequisite for this to occur is the presence of substantial momentum-dependent components in Γ (see below). For a long time, this problem was studied only under the assumption that the new Fermi surface becomes multi-connected, while the occupation numbers $n(\mathbf{p})$ remain 1 or 0 [18]. However, the expansion (1) can also be applied to evaluate variations of E_0 when continuous *quasiparticle* distributions $n(\mathbf{p})$ come into play, and beyond the point where the NCS is violated, it often turns out that such unusual for SFLT, but accustomed e.g. for the Bogoliubov theory of superconductivity, distributions yield a lower energy than any discrete-valued solution with multi-connected Fermi surface. This compels us to treat the quasiparticle occupation numbers $n(\mathbf{p})$ in Eq. (1) as variational parameters and look for a new location of the minimum of $E(n)$ from the variational condition [7]

$$\frac{\delta E_0(n)}{\delta n(\mathbf{p})} = \mu \ , \qquad \mathbf{p} \in \mathrm{C} \ , \tag{2}$$

with δE_0 given by Eq. (1). The region C is the place of the residence of the fermion condensate, since the l.h.s. of Eq. (2) is just the quasiparticle energy $\epsilon(\mathbf{p}; n)$, and Eq. (2) is equivalent to $\epsilon(\mathbf{p} \in \mathrm{C}; n) = \mu$. Outside this domain, whose boundaries are determined from Eq. (2) itself, the old solution $n_F(\mathbf{p})$ survives.

Similar results can be obtained by summation of diagrams of the mass operator in Dyson equation [17]. The transition occurs, if in this equation, there exists a bifurcation point. Beyond this point, the ground state distribution $n(\mathbf{p})$ is evaluated from relation

$$\epsilon^0_{\mathbf{p}} + \Sigma(\mathbf{p}, \varepsilon = 0; n) = \mu \ , \qquad \mathbf{p} \in \mathrm{C} \ , \tag{3}$$

which is equivalent to the equation $\epsilon(\mathbf{p} \in \mathrm{C}; n) = \mu$. If one neglects for a while the ε-dependence of $\Sigma(\mathbf{p}, \varepsilon)$, then the FC Green function $G(\mathbf{p} \in \mathrm{C}, \varepsilon) = [\varepsilon + \mu - \epsilon^0_{\mathbf{p}} - \Sigma(\mathbf{p} \in \mathrm{C}, \varepsilon)]^{-1}$ becomes $1/\varepsilon$, that leads to a change of the Fermi liquid topological charge N [8]. Recall that for ordinary homogeneous Fermi liquids, with $G(p, \varepsilon) \sim [\varepsilon - \epsilon(p) + \mu]^{-1}$, and for Luttinger liquids as well, the value of N is unity, as given by the integral

$$N = \oint\limits_L \frac{dl}{2\pi i}\, G(\mathbf{p}, \varepsilon{=}i\Omega)\, \partial_l G^{-1}(\mathbf{p}, \varepsilon{=}i\Omega)\ , \tag{4}$$

taken along the contour L, embracing the Fermi line in the 3D space (p_x, p_y, Ω). However, upon inserting $G_{\mathrm{C}}(p, \varepsilon) = 1/\varepsilon$ into the integral (4), one finds $N = 1/2$, implying that systems with the FC form a separate class of normal Fermi liquids [8]. As we shall see, this result holds if the energy dependence of the mass operator is incorporated.

The substantial momentum dependence of Γ, needed for solution of Eqs. (2) and (3) to exist, emerges, for example, in the vicinity of an impending antiferromagnetic phase transition. In this case, the static spin susceptibility $\chi(\mathbf{q}, \omega = 0) \sim [\beta^2 + \kappa^2(\mathbf{q} - \mathbf{Q})^2]^{-1}$ [19] diverges at $\mathbf{q} \to \mathbf{Q} = (\pi, \pi)$, since β vanishes at the transition point. The direct term Γ_d in Γ, being proportional to $-\chi(\mathbf{q}, \omega = 0)$, diverges as well, and so does the exchange term $\Gamma_e(\mathbf{p}, \mathbf{p}_1, \mathbf{q}) \sim [\beta^2 + \kappa^2(\mathbf{p}_1 - \mathbf{p}_2 + \mathbf{q} - \mathbf{Q})^2]^{-1}$ because of the antisymmetry relations imposed on Γ [20]. It is the exchange term, taken at $\mathbf{q} = 0$, that enters Eq. (1). Parameters β and κ are assumed to depend on the density ρ rather than on $n_F(\mathbf{p})$ itself, so that $\delta\Gamma_e/\delta n_F(\mathbf{p}) = 0$. If the amplitude Γ is approximated by Γ_e, then upon inserting Eq. (1) into Eq. (2) the latter is recast to $\epsilon_1(\mathbf{p}; n_F) + \sum\limits_{\mathbf{p}_1} \Gamma_e(\mathbf{p}, \mathbf{p}_1; n_F)\, n(\mathbf{p}_1) = \mu$. From the definition $\epsilon_1(\mathbf{p}) = \epsilon(\mathbf{p}; n_F) - \sum\limits_{\mathbf{p}_1} \Gamma_e(\mathbf{p}, \mathbf{p}_1) n_F(\mathbf{p}_1)$, it is clear that the variational derivative $\delta\epsilon_1(\mathbf{p})/\delta n_F(\mathbf{p})$ vanishes. Therefore $\epsilon_1(\mathbf{p})$ is reduced to the LDA spectrum $\epsilon^0_{\mathbf{p}}$, and Eq. (2) takes the form

$$\epsilon^0_{\mathbf{p}} + \sum_{\mathbf{p}_1} \Gamma_e(\mathbf{p}, \mathbf{p}_1)\, n(\mathbf{p}_1) = \mu\ , \qquad \mathbf{p} \in \mathrm{C} \tag{5}$$

often employed in theory of fermion condensation. It is worth noting that Eq. (5) can be derived from the effective energy functional $E_{\mathrm{eff}}(n) = \sum\limits_{\mathbf{p}} \epsilon^0_{\mathbf{p}}\, n(\mathbf{p}) + 1/2 \sum\limits_{\mathbf{p}\mathbf{p}_1} \Gamma_e(\mathbf{p}, \mathbf{p}_1)\, n(\mathbf{p})\, n(\mathbf{p}_1)$, which is appropriate only for the rearrangement problem and has no connection with the Hartree-Fock part of the ground-state energy functional.

Results of numerical calculations of Eq. (5), which are insensitive to the choice of the parameter κ, show that a FC arises when the filling approaches 1/2. Once it appears, the FC resides close to the vHP. As seen from Fig. 1, the fraction η of the Brillouin zone occupied by the FC remains rather small, attaining a maximum ~ 0.1 when the filling slightly exceeds 1/2, and the hole pocket is centered around (π, π) (for more details, see [17]).

The degeneracy of the SP spectrum at $T = 0$, a salient feature of the solution given by Eqs. (2), (3), is lifted by pairing interactions which are not included into Eq. (1) [7,17]. In doing so, the BCS occupation numbers $v^2(\mathbf{p})$ coincide with $n(\mathbf{p})$ evaluated from Eq. (2) or Eq. (3) provided the BCS coupling constant is rather small. Evidently, in obtaining both these distributions damping effects can be ignored. For this reason, superfluid systems with and without the FC look more alike than normal ones, since in normal states of conventional Fermi liquids the damping makes no difference, whereas in normal states of systems with the FC, the damping becomes a real "weathermaker". Indeed, the relevant contribution to $\mathrm{Im}\,\Sigma_R(\mathbf{p}, \varepsilon)$ is given by [21]

$$\mathrm{Im}\,\Sigma_R(\mathbf{p}, \varepsilon) \sim \sum_{\mathbf{q}, \mathbf{p}_1} \iint d\omega\, d\varepsilon_1 F(\varepsilon, \omega, \varepsilon_1, T)\, |\Gamma(\mathbf{p}, \varepsilon, \mathbf{p}_1, \varepsilon_1, \mathbf{q}, \omega; n)|^2$$
$$\times\ \mathrm{Im}\,G_R(\mathbf{p} - \mathbf{q}, \varepsilon - \omega)\, \mathrm{Im}\,G_R(-\mathbf{p}_1, -\varepsilon_1)\, \mathrm{Im}\,G_R(\mathbf{q} - \mathbf{p}_1, \omega - \varepsilon_1)\ , \qquad (6)$$

where the factor $F(\varepsilon, \omega, \varepsilon_1, T) = \cosh(\frac{\varepsilon}{2T})[\cosh(\frac{\varepsilon_1}{2T}) \cosh(\frac{\varepsilon - \omega}{2T}) \cosh(\frac{\omega - \varepsilon_1}{2T})]^{-1}$, $|\Gamma|^2$ is the absolute square of the scattering amplitude, properly averaged over spin variables, and G_R is the retarded Green function with $\mathrm{Im}\,G_R(\mathbf{p}, \varepsilon) = -\gamma(\mathbf{p}, \varepsilon)[(\varepsilon - \sigma(\mathbf{p}, \varepsilon) - s(\mathbf{p}))^2 + \gamma^2(\mathbf{p}, \varepsilon)]^{-1}$, where $\sigma(\mathbf{p}, \varepsilon) = \mathrm{Re}\,\Sigma_R(\mathbf{p}, \varepsilon) - \mathrm{Re}\,\Sigma_R(\mathbf{p}, \varepsilon = 0)$ and $s(\mathbf{p}) = \epsilon^0_{\mathbf{p}} + \mathrm{Re}\,\Sigma_R(\mathbf{p}, \varepsilon = 0) - \mu$.

We restrict ourselves to temperatures T markedly lower than the characteristic temperature T_f of destroying the FC that allows one to ignore the T-dependence of η. To simplify the problem, we replace the function $\gamma(\mathbf{p}, \varepsilon)$ by a set of functions of the single variable ε, e.g. in the FC region, $\gamma(\mathbf{p} \in \mathrm{C}, \varepsilon)$ is reduced to $\gamma_{\mathrm{C}}(\varepsilon)$. The complementary region of momentum space, in which the dispersion of the spectrum $\epsilon(\mathbf{p})$ has a nonzero value, is composed of two subregions. The first, adjacent to the FC domain and henceforth denoted by T, is a transition region, in which the same decay processes, as in the FC region, are still kinematically

allowed. The second subregion, denoted by M, is located around diagonals of the Brillouin zone. Here some of these processes are either kinematically forbidden or at least strongly suppressed. Correspondingly, $\gamma(\mathbf{p} \in \mathrm{T}, \varepsilon) \to \gamma_{\mathrm{T}}(\varepsilon), \gamma(\mathbf{p} \in \mathrm{M}, \varepsilon) \to \gamma_{\mathrm{M}}(\varepsilon)$. To close the set of equations of the problem, the amplitude Γ should somehow be specified. Bearing in mind that η is small, we shall initially neglect the FC contributions to Γ, replacing it by $\Gamma(n_F)$.

We start with the case $\varepsilon \gg T$, and set $T = 0$ in the integral (6), thus dropping all T-dependent contributions. First we evaluate $\gamma_{\mathrm{C}}(\varepsilon \to 0)$. In this case, (i) contributions from processes involving only the FC states prevail (see below), (ii) the quantity $|\Gamma(n_F)|^2 \sim \beta^{-4}$ can be factored out of the integral (6), and (iii) the quantity $s(\mathbf{p} \in \mathrm{C}, T)$, which vanishes over the FC region at $T = 0$, can be verified to remain small compared to leading terms in Eq. (6), and thus can be neglected. As a result, the energy and momentum integrations in Eq. (6) separate. Taking for certainty $\varepsilon > 0$ and omitting numerical factors, we are left with

$$\gamma_{\mathrm{C}}(\varepsilon \to 0) \sim \beta^{-4}\eta^2 \int_0^{\varepsilon}\int_0^{\omega} A_{\mathrm{C}}(\varepsilon - \omega)\, A_{\mathrm{C}}(-\varepsilon_1)\, A_{\mathrm{C}}(\omega - \varepsilon_1)\, d\omega\, d\varepsilon_1 \ , \tag{7}$$

where $A_{\mathrm{C}}(\varepsilon) = \mathrm{Im}\, G_R(\mathbf{p} \in \mathrm{C}, \varepsilon)$. To proceed, we insert $\gamma_{\mathrm{C}}(\varepsilon \to 0) \sim \varepsilon^{\nu_{\mathrm{C}}}$ into the Kramers-Krönig relation to obtain $\sigma_{\mathrm{C}}(\varepsilon \to 0) \sim \varepsilon^{\nu_{\mathrm{C}}}$. We then substitute γ_{C} and σ_{C} into A_{C} and find $A_{\mathrm{C}}(\varepsilon \to 0) \sim \varepsilon^{-\nu_{\mathrm{C}}}$. Finally, upon inserting this result into Eq. (7), we arrive at $\nu_{\mathrm{C}} = 1/2$ [15]. More precisely, one obtains $\gamma_{\mathrm{C}}(\varepsilon \to 0) \sim \beta^{-1}(\eta\varepsilon_F^0\varepsilon)^{1/2}$ and

$$G_R(\mathbf{p} \in \mathrm{C}, \varepsilon \to 0) \sim e^{-i\pi/4}[\gamma_{\mathrm{C}}(\varepsilon \to 0)]^{-1} \sim e^{-i\pi/4}\beta(\eta\varepsilon_F^0\varepsilon)^{-1/2} \ . \tag{8}$$

This result can be shown to hold even if the momentum dependence of the quantities $\gamma_{\mathrm{C}}(\mathbf{p}, \varepsilon)$ and $\sigma_{\mathrm{C}}(\mathbf{p}, \varepsilon)$ is properly taken into account. We see that in the FC region, the conventional structure of the Green function is destroyed, the familiar pole being replaced by a branch point at $\varepsilon = 0$. What happens to the topological charge N? Upon inserting the Green function (8) into Eq. (4) and performing simple integration, we are again led to Volovik's previous result $N = 1/2$ [8], in spite of the dramatic alteration of the Green function itself that occurs in the FC region.

In the transition region T, the decay into the FC states is not kinematically forbidden. Accordingly, $\gamma_{\mathrm{T}}(\varepsilon \to 0) \sim \beta^{-1}(\eta\varepsilon_F^0\varepsilon)^{1/2}$, while the function $s(\mathbf{p} \in \mathrm{T})$ already differs from

zero. Requiring it to vanish at the boundaries of the FC region along with its first derivative, one finds that in the region T, the conventional structure of the Green function is recovered, but in the vicinity of the FC domain, single-particle excitations appear to be ill-defined, since the pole of $G(\mathbf{p},\varepsilon)$ is located close to the imaginary energy axis.

In the region M, dominant contributions to $\mathrm{Im}\,\Sigma_R(\varepsilon)$ come from a process associated with the generation of three states: two from the FC region and one from the M region. In this case, the formula for finding $\gamma_{\rm M}(\varepsilon\to 0)$ reads

$$\gamma_{\rm M}(\varepsilon{\to}0) \sim \beta^{-4} \sum_{\mathbf{p},\mathbf{p}_1} \int_0^{\varepsilon}\!\!\int_0^{\omega} A_{\rm C}(-\varepsilon_1)\, A_{\rm C}(\omega{-}\varepsilon_1)\, [1-\theta(\mathbf{p})]\, P_{\rm C}(\mathbf{p}{-}\mathbf{p}_1)\, A_{\rm M}(\mathbf{p},\varepsilon{-}\omega)\, d\omega\, d\varepsilon_1 \ , \quad (9)$$

where $P_{\rm C}(\mathbf{q}) = \sum_{\mathbf{p}} \theta(\mathbf{p})\theta(\mathbf{p}-\mathbf{q})$ and $\theta(\mathbf{p}) = 1$ if $\mathbf{p}\in$ C and otherwise vanishes. It is seen that in this case, the momentum and energy integrations do not separate. However, one can take advantage of the fact that the spectrum $\xi_{\rm M}(\mathbf{p}) = \epsilon_{\rm M}(\mathbf{p}) - \mu$ is proportional to $(p-p_F)$ and introduce $\xi_{\rm M}(\mathbf{p})$ as a new variable. Then after simple integration, we are led to the result $\nu_{\rm M} = 1$ postulated in the model of a marginal Fermi liquid (MFL) [22]. Evaluation of the η-dependence of relevant quantities in the M region yields $\gamma_{\rm M}(\varepsilon\to 0) \sim \beta^{-2}\eta^{1/2}\varepsilon$ and $\sigma_{\rm M}(\varepsilon\to 0) \sim \beta^{-2}\eta^{1/2}\varepsilon\ln|\varepsilon|$.

These results can be applied to the case $\varepsilon \sim T$, where according to Eq. (8), the leading term in the FC Green function has the form $G_R(\mathbf{p}\in \mathrm{C},\varepsilon) \sim e^{-i\pi/4}[\gamma_{\rm C}(\varepsilon,T)]^{-1}$. Upon inserting this expression into Eq. (7) one finds that the damping $\gamma_{\rm C}(x,T)$, where $x=\varepsilon/T$, can be displayed as $\gamma_{\rm C}(x,T) = \gamma_{\rm C}\sqrt{T\varepsilon_F^0}\, D(x)$ where the constant $\gamma_{\rm C}$ specifies the compound, while the dimensionless quantity $D(x)$ obeys the integral equation

$$D(x) = \cosh\frac{x}{2}\iint \frac{dy\,dz}{\cosh\frac{y}{2}\cosh\frac{x-z}{2}\cosh\frac{z-y}{2}\, D(-y)\, D(x{-}z)\, D(z{-}y)} \ . \quad (10)$$

With this result, the damping $\gamma_{\rm M}(x,T)$ is calculated straifghtforwardly:

$$\gamma_{\rm M}(x,T) = \gamma_{\rm M} T\cosh\frac{x}{2}\iint \frac{dy\,dz}{\cosh\frac{y}{2}\cosh\frac{x-z}{2}\cosh\frac{z-y}{2}\ D(-y)\, D(z{-}y)} \ , \quad (11)$$

the constant $\gamma_{\rm M}$ being a characteristics of the given compound. The function $\gamma_{\rm M}(x,T)/T$ starts out of the origin as a parabolic function $\gamma_{\rm M}(x,T)/T \sim 1 + 0.1\,x^2$. The asymptotic regime $\gamma_{\rm M}(x,T)/T \sim x$, stemming from Eq. (9), begins at $x\sim 2.5$.

In normal states with the nonzero value of the pseudogap, relations (10), (11) hold, as long as it remains less than the damping $\gamma_{\rm C}(T)$. These relations persist even in superfluid states provided the gap value meets the same restriction. On the other hand, they are violated if energy attains values, at which contributions to $\gamma(\varepsilon)$ that were omitted from Eqs. (7) and (9) become comparable to the terms that were retained. A leading correction $\delta\gamma_{\rm C}(\varepsilon)$ to the integral (7) comes from final states, that involve one hole (particle) belonging to the region T. Eq. (9) can be employed to estimate this contribution, with the single replacement $s(\mathbf{p} \in {\rm M}) \to s(\mathbf{p} \in {\rm T})$. We find $\delta\gamma_{\rm C}(\varepsilon \to 0) \sim \beta^{-1}\varepsilon$, which is independent of the η value. Estimation of other corrections to $\gamma(\varepsilon)$ is carried out along the same lines, justifying the identification of (7) and (9) as paramount contributions to ${\rm Im}\,\Sigma_R(\mathbf{p}, \varepsilon \to 0)$ until ε exceeds the characteristic FC energy $\varepsilon_{FC} \simeq \eta\varepsilon_F^0$, evaluated by comparison of $\delta\gamma_{\rm C}(\varepsilon)$ and $\gamma_{\rm C}(\varepsilon) \sim \beta^{-1}(\eta\varepsilon_F^0\varepsilon)^{1/2}$.

At energies $\varepsilon \geq \varepsilon_{FC}$, the corrections exhibit themselves in full force, so Eq. (6) should be solved numerically in conjunction with the Kramers-Krönig relation, employed to connect $\gamma(\varepsilon)$ and $\sigma(\varepsilon)$. This is done with the aid of an iteration procedure, which converges rapidly. In Fig. 2 we display results from these calculations carried out with the same amplitude Γ that was employed in the calculations of the FC characteristics shown in Fig. 1. Two different η values, specifying the fraction of the Brillouin zone occupied by the FC, were considered: (a) $\eta = 0.1$, close to the maximum η value in the model of fermion condensation driven by antiferromagnetic fluctuations [17], and (b) $\eta = 0.01$. In spite of the simplicity of the interaction adopted, salient features of ARPES data [3,6,14] are reproduced, including the universal behavior of the ratio ${\rm Im}\,\Sigma_R(\mathbf{p} \in {\rm M}, \varepsilon, T; \eta)/T$ as a function of $x = \varepsilon/T$, uncovered in [3]. Moreover, our theory predicts the same behavior of ${\rm Im}\,\Sigma_R(\mathbf{p} \in {\rm M}, \varepsilon, T; \eta)/T$ for different compounds provided results are properly normalized. We choose this normalization to ensure the same slope $k(\eta) = |\partial\,{\rm Im}\,\Sigma_R(\mathbf{p} \in {\rm M}, \varepsilon, T; \eta)/\partial\varepsilon|$ at $x \gg 1$ where the damping changes linearly with ε. The latter is demonstrated in Fig. 3, where two functions $|{\rm Im}\,\Sigma_R(\mathbf{p} \in {\rm M}, \varepsilon, T; \eta)|/T$, evaluated at $\eta = 0.1$ and $\eta = 0.01$ and normalized to the same slope $k = 0.75$, are compared to data measured in [3] for the optimally doped cuprate $Bi_2Sr_2CaCu_2O_{8+\delta}$. At

the same time as follows from Eq. (10), in the FC region the above universal scaling of the ratio $\mathrm{Im}\,\Sigma_R(\varepsilon,T)/T$ is destroyed, and instead $\mathrm{Im}\,\Sigma_R(\mathbf{p}\in \mathrm{C},\varepsilon,T)$ displays $\sqrt{T}$-dependence at $x\leq 1$.

The above scenario in which the fermion condensation precedes the antiferromagnetic phase transition does apply in the three-dimensional case, although the range of the FC region shrinks markedly. Along the same lines, one can analyze the situation with fermion condensation in the vicinity of other second order phase transitions, such as charge-density-wave instability. So far the feedback of the FC on the scattering amplitude Γ has been ignored. However, the simplest FC diagram, i.e. a loop, evaluated with the FC Green function (8), diverges logarithmically. As a result, we are led to a familiar problem of the parquet-diagram summation, solution of which will be reported in a separate paper.

Summing up the results of our analysis, we infer that infinite electron systems with a fermion condensate, independently of the dimensionality, do not admit Landau quasiparticles, since the renormalization factor $z=(1-\partial\Sigma/\partial\varepsilon)^{-1}_F$ that determines the quasiparticle weight in the single-particle state vanishes in all the regions of the Brillouin zone. In the FC domain, the value of the topological charge N is found to be $N=1/2$ indicating that systems with a fermion condensate form a separate class of Fermi liquids. The model of fermion condensation presented here allows one to explain basic features of the spectral functions of normal states of high-T_c superconductors, including the MFL behavior of the damping of SP states around the diagonals of the Brillouin zone (the M region) and the suppression of the peaks in APRES data in the immediate vicinity of the van Hove points. And the universal behavior of the ratio $\mathrm{Im}\,\Sigma_R(\mathbf{p}\in \mathrm{M},x,T)/T$ as a function of $x=\varepsilon/T$ established in [3] for the M region is also reproduced in this model. Moreover, when properly normalized, our results for $\mathrm{Im}\,\Sigma_R(\mathbf{p}\in \mathrm{M},x,T)/T$ calculated for different compounds collapse on the same curve. However, as we have seen, this universal behavior is destroyed in the C region close to the van Hove points.

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FIGURES

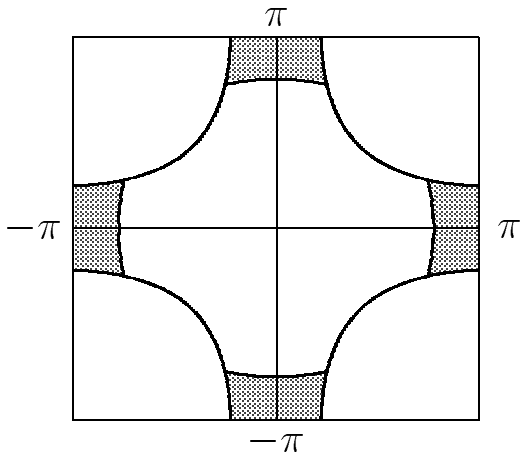

FIG. 1. The Fermi surface in the model of fermion condensation, driven by antiferromagnetic fluctuations with the scattering amplitude $\Gamma(\mathbf{p},\mathbf{p}_1) = (N(0))^{-1}[\beta^2 + \kappa^2(\mathbf{p}-\mathbf{p}_1-\mathbf{Q})^2]^{-1}$, where $N(0)$ is the density of states and $\beta = 0.2$. The regions occupied by FC are shaded.

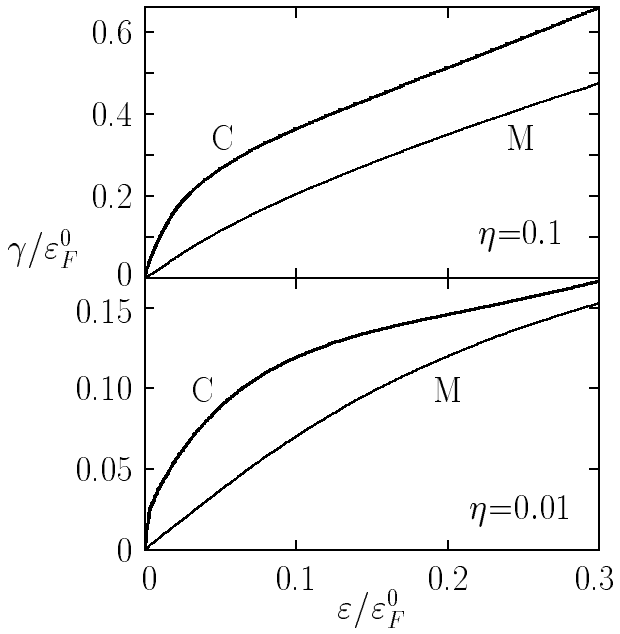

FIG. 2. The damping of the single-particle states at $T = 0$ in the vicinity of the vHP (solid lines) and around the diagonals of the Brillouin zone (thin lines), calculated with the same scattering amplitude Γ for $\eta = 0.1$ (upper panel) and $\eta = 0.01$ (lower panel), and measured in ε_F^0.

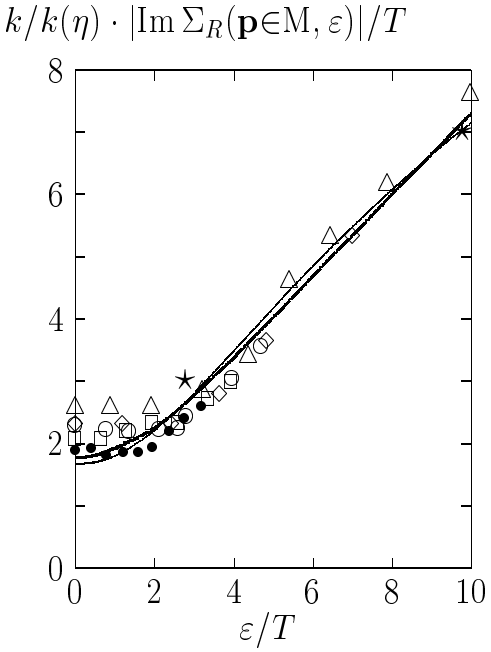

FIG. 3. The ratio $|\mathrm{Im}\,\Sigma_R(\varepsilon)|/T$ around the diagonal of the Brillouin zone as a function of ε/T, calculated for $\eta = 0.1$ (solid line) $\eta = 0.01$ (thin line) and normalized to the same slope $k = 0.75$, i.e. multiplied by the factor $k/k(\eta)$. The experimental data for the optimally doped cuprate $Bi_2Sr_2CaCu_2O_{8+\delta}$ [3] are shown by open and solid circles ($T = 300\,\mathrm{K}$), triangles and squares ($T = 90\,\mathrm{K}$), and diamonds and stars ($T = 48\,\mathrm{K}$).